Parity doubling from Weinberg sum rules

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We investigate the relation among slopes and intercepts of Regge trajectories for mesons of a given spin and different parities using large N_c arguments and the matching to perturbative QCD in the deep-Minkowski region. For spin-1 mesons of opposite parities we prove that: a) for large and increasing N_c, the scale $\Lambda^{(V,A)}$ separating the resonance-dominated and the perturbative-saturated region in the channels V, A grows as $\sqrt{N_c}$; b) to satisfy the Weinberg sum rules the slopes of Regge trajectories for mesons of opposite parities must coincide; c) their intercepts may differ and their difference corresponds to the difference between Λ^V and Λ^A. Some arguments indicate that this difference should tend to zero as $N_c \to \infty$.

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1. Recently the issue as to whether radial excitations of mesons with a given spin but of opposite parities become eventually degenerate in mass in the large N_c limit has been hotly debated [1]–[17]. Different works have clashed as to whether chiral symmetry restoration at high energies implies that meson masses asymptotically approach each other [1]–[10], or, on the contrary, the footprint of chiral symmetry breaking persists for arbitrarily high mass mesons (in the large N_c limit) [11]–[14]. Some results based on AdS/QCD correspondence [15]–[16] have also questioned whether the slopes of Regge trajectories for mesons of opposite parities should be equal. While this issue has a long history (reviewed recently in [7, 17, 18]), the increasing interest has been fuelled by the latest improvements in meson phenomenology [18]–[21]. In this letter we study the possibility to relate slopes and intercepts of Regge trajectories for mesons of a given spin and opposite parities by making a careful use of Weinberg sum rules [22, 23].

This issue has been studied traditionally by considering the $N_c \to \infty$ limit at the outset and using the Operator Product Expansion (OPE) in the Euclidean region. The main point of this letter is that keeping N_c large but finite is useful to keep under theoretical control the "crossover" between the region of resonance saturation and the high energy region where perturbative QCD is valid. Also we shall work consistently in Minkowski momentum space throughout, as opposed to previous analysis.

Let us consider the correlators

$$\begin{aligned}
&\Pi^j_{\mu\nu}(x,y) = \langle T\left(J_\mu(x) J_\nu(y)\right)\rangle \\
&\equiv (-i)\int \frac{d^4p}{(2\pi)^4}\exp[ip(y-x)]\Pi^j_{\mu\nu}(p^2), \qquad (1)\\
&j = V,\, A; \quad J_\mu = \Big(\bar q(x)\gamma_\mu q(x),\, \bar q(x)\gamma_5\gamma_\mu q(x)\Big).
\end{aligned}$$

As we keep the chiral limit in the major part of our analysis we do not need to specify the internal symmetry group and omit the flavor indices. The color degrees of freedom of the quark fields $\bar q, q$ are also omitted in the notation.

For finite N_c two different, non-overlapping, regions of physics can be clearly identified in the physical (Minkowski) momentum region: a region dominated by resonances over a non-resonant background, and a region where perturbative QCD is reliable. This clear separation is due to chiral symmetry breaking and confinement in the QCD vacuum, on the one side, and to the asymptotic freedom of QCD, on the other side. In addition, there is an intermediate region where neither resonance dominance or perturbative physics describe well the data; resonances are nearly invisible in the continuum of multiparticle contributions and perturbation theory becomes unreliable. We note that large N_c counting rules indicate that the multiparticle background must disappear in the large N_c limit.

At this point we need to be more definite about how we count resonances and in order to do this we introduce some 'error bar' in the magnitude of the correlators $|\Delta\Pi/\Pi| \sim \epsilon$. Resonances of (relative) height lower than ϵ over the background will be counted as part of the continuum, whereas those that stand out higher than ϵ will be retained. The quantity ϵ will be universal for both correlators. For given ϵ and N_c one can find a finite number of visible resonances and establish an upper bound $p^2 \leq \Lambda_R^2$ above which one deals with continuum generated by intermediate multiparticle states but not resolved into resonances.

From the other end, at high energies one expects quark-hadron duality to hold [24] and perturbation theory to provide accurate predictions with a (relative) precision ϵ down to a scale Λ^2_{PT}. By construction, $\Lambda_R < \Lambda_{PT}$. At intermediate values $\Lambda_R^2 < p^2 < \Lambda^2_{PT}$ the non-resonant multihadron picture is adequate.

2. Now let us increase the number of colors. According to the usual large N_c counting rules we expect that: (a) Resonances become narrower and more distinct showing clearer Breit-Wigner shapes and increasing their mag-

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nitudes. Their position, on the contrary, are independent of N_c at leading order. (b) At a given value of p^2 the number of possible intermediate multiparticle states is fixed, but their coupling constants behave as inverse powers of N_c and consequently the nonresonant hadron background at fixed p^2 decreases. Then for a fixed value of ϵ more resonances become visible as we increase the number of colors, $\Lambda_R(N_c) \leq \Lambda_R(N_c'), \quad N_c < N_c'$. (c) The non resonant background due to multiparticle states decreases. This background is lower at lower values of p^2 due to phase space considerations.

Let us imagine drawing a band of (relative) width ϵ around the perturbative prediction. At low values of p^2, perturbation theory fails of course badly in describing the two point function because resonances are totally beyond the scope of perturbation theory. As we move to larger values of p^2, the non-resonant background grows and resonances become broader due to phase space considerations; eventually they all disappear within the 'error bar' band, thus merging in a continuum. However, this continuum does not necessarily agree with the one predicted by perturbation theory if the value of p^2 is too low. As we increase N_c, the resonances become more marked and more and more of them become visible at a given value of ϵ and the value of $(\Lambda_R)^2$ increasing with the number of resonances included. The non-resonant, non-perturbative background decreases furthermore as N_c increases. Correspondingly, at the values of p^2 where resonances disappear into the continuum, perturbation theory becomes more and more reliable.

We shall assume a Regge-like behavior for the radially excited states in the different channels. Linearly rising trajectories imply that Λ_R^2 grows linearly with the number of visible resonances. We shall see below that large N_c counting rules imply that the number of visible resonances increases linearly with N_c, so the region of validity of perturbation theory is reached rather quickly. Combined with the disappearance of the non-perturbative background at large N_c, it is rather clear that for any value of ϵ there should be a value of N_c large enough (but still finite) where $\Lambda_R \simeq \Lambda_{PT}$.

If we accept this highly plausible conjecture, we can, with an error bounded by ϵ, replace $\Pi^j_{\mu\nu}(p^2)$ by $\Pi^{j,PT}_{\mu\nu}(p^2)$ for values of p^2 beyond the last visible resonance. Whereas it is clear that none of vector resonances saturating the VV correlator in (1) must necessarily coincide in mass with the AA one, the chiral symmetry of QCD guarantees the coincidence up to non-perturbative corrections of order $1/(p^2)^3$ of VV and AA correlators at very high momenta. In fact, the non-perturbative corrections, that are suppressed by inverse powers of momenta, are also suppressed by powers of N_c as $p^2 \geq \Lambda^2_{PT} \simeq \Lambda^2_R \sim N_c$. Thus, at leading order in N_c, $\Pi^{j,PT}_{\mu\nu}(p^2)$ is actually identical for the V and A channels.

Notice that the OPE, valid in the deep Euclidean region, implies in turn via dispersion relations corrections to $\Pi^{j,PT}_{\mu\nu}(p^2)$ proportional to the four-quark condensate in the physical Minkowski region. On dimensional grounds these corrections are down by a power of $1/(p^2)^3$ and in the kinematic region we are considering are of order $1/N_c$, to be compared with the leading perturbative contribution of order N_c^2 (assuming again that $(\Lambda^j_R)^2 \sim N_c$). Thus the difference between the V and A channels is much suppressed in the large N_c limit in the region of transition between resonance domination and perturbation theory.

Λ^V_R need not be equal to Λ^A_R owing to chiral symmetry breaking. However, taking into account the previous arguments we can conservatively assume that the difference is of $\mathcal{O}(1)$, although it is probably even smaller. Assuming that $(\Lambda^V_R)^2 - (\Lambda^A_R)^2 \sim \mathcal{O}(1)$ is sufficient for our purposes.

The number of visible resonances need not be strictly the same in both channels either. Let N^V and N^A be the numbers of such resonances (visible with precision ϵ) for a given N_c. If linear Regge trajectories are appropriate for large meson masses, $(m^j_n)^2 \simeq (m^j_0)^2 + a^j n$, $n \gg 1$, then evidently $N^j \sim (\Lambda^j_R)^2/a^j$ and increases with growing N_c.

3. We shall now make use of Weinberg sum rules [22, 23]. Let us decompose the correlators in spin zero and spin one components

$$\Pi^j_{\mu\nu}(p^2) = \Big(- g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}\Big)\Pi^j_1(p^2) + \frac{p_\mu p_\nu}{p^2}\Pi^j_0(p^2), \quad (2)$$

and then use the spectral representation

$$\Pi^j_1(p^2) = -\int_0^\infty ds \frac{\rho^j(s)}{p^2 - s + i\varepsilon},\ \rho^j(s) = \frac{1}{\pi}\mathrm{Im}\Pi^j_1(s) > 0, \quad (3)$$

where $\rho^j(s)$ is related to the probability of producing particles with invariant mass squared s. As to the longitudinal projection one has to remove a possible massless pole in the vector channel and reproduce the pion pole in the axial one

$$\Pi^V_0(0) = -\Pi^V_1(0) = \int_0^\infty ds \frac{\rho^V(s)}{s},$$

$$\Pi^A_0(0) = F^2_\pi - \Pi^A_1(0) = F^2_\pi + \int_0^\infty ds \frac{\rho^A(s)}{s}. \quad (4)$$

The current conservation in (1) for $x \neq y$ is compatible with constant $\Pi^j_0(p^2) = \Pi^j_0(0)$.

It is well known that both the spectral representation (3) and the spectral integrals (4) are formal, being UV divergent as the probability $\rho^j(s)$ does not decrease at very large s, being eventually saturated by the imaginary part of the perturbative decay amplitude into quarks which increases linearly with s. As to the IR pole $1/s$ the absence of other massless particles but the pion and the Adler zeroes in the chiral limit guarantee the IR integrability of $\rho^j(s)$. Thus the dispersion relations need subtractions of the short-distance singularities. On the other hand, the Wilson analysis of OPE for correlators in x space allows

to locate the singularities on the light cone which are perturbative due to asymptotic freedom and equivalent for vector and axial-vector channels. Owing to this fact one can combine the difference of VV and AA correlators to eliminate those singularities and derive two well convergent Weinberg sum rules [22, 23],

$$\int_0^\infty ds \frac{\rho^V(s) - \rho^A(s)}{s} = F_\pi^2, \tag{5}$$

$$\int_0^\infty ds \Big(\rho^V(s) - \rho^A(s)\Big) = 0. \tag{6}$$

We now consider these sum rules for a finite but large value of N_c and assume that $\Lambda_R^V \geq \Lambda_R^A$ (the reverse case can be treated similarly and leads to the same results). Let us saturate the entire spectral density $\rho^V(s) - \rho^A(s)$ by well-separated resonances up to $s = (\Lambda^A)^2$, by resonances for $\rho^V(s)$ and by perturbation theory for $\rho^A(s)$ when $(\Lambda^A)^2 < s < (\Lambda^V)^2$, as well as by perturbation theory for $\rho^{V,A}(s)$ when $(\Lambda^V)^2 \leq s$ with

$$\rho_{PT}(s) = N_c C_0 s, \quad C_0 \equiv \frac{1}{24\pi^2}\Big(1 + \frac{N_c\alpha_s}{3\pi} + \dots\Big). \tag{7}$$

As previously indicated, the contribution from the condensates can be safely neglected if $(\Lambda_R^{(V,A)})^2$ is proportional to N_c. Then the Weinberg sum rules (5) and (6) read

$$\sum_{n=0}^{N^V} (F_n^V)^2 - \sum_{n=0}^{N^A} (F_n^A)^2 = F_\pi^2 + N_c C_0 \left((\Lambda^V)^2 - (\Lambda^A)^2\right), \tag{8}$$

$$\sum_{n=0}^{N^V} (F_n^V)^2 (m_n^V)^2 - \sum_{n=0}^{N^A} (F_n^A)^2 (m_n^A)^2 = \frac{1}{2} N_c C_0 \Big((\Lambda^V)^4 - (\Lambda^A)^4\Big), \tag{9}$$

where we have used the fact that for separated narrow Breit-Wigner resonances one can calculate their individual contributions

$$\pi\rho_n^j(s) = \frac{(F_n^j m_n^j)^2\ \Gamma_n^j m_n^j}{\Big(s - (m_n^j)^2\Big)^2 + (\Gamma_n^j m_n^j)^2}, \tag{10}$$

extrapolating the integration to infinity and the result is independent of the width.

4. At larger N_c one observes the narrowing and growing of resonances, but they become progressively less marked at higher values of $(m_n^j)^2$. Resonances become invisible when the resonance width $m_n^j \Gamma_n^j$ becomes comparable with the distance between neighbor resonances (see similar arguments in [25]). For linear trajectories, in the Regge description of mesons [1], $\Gamma_n^j \sim B^j m_n^j / N_c$. Thus resonances in a given channel overlap when their widths $m_n^j \Gamma_n^j$ are equal to the corresponding slopes $m_n^j \Gamma_n^j \sim B^j (m_n^j)^2 / N_c \sim a^j$, *i.e.* for $N^j \sim N_c / B^j$. It corresponds to $(\Lambda_R^j)^2 \sim N^j a^j \sim N_c a^j / B^j$, showing that the number of visible resonances in each channel is proportional to N_c as previously indicated. The corresponding maxima are given by

$$\pi\rho_n^j(s)\Big|_{s=(m_n^j)^2} = \frac{(F_n^j)^2 m_n^j}{\Gamma_n^j}, \tag{11}$$

At the point where resonances become invisible (at a fixed value of ϵ), the spectral density levels off at a value

$$\pi\rho_n^j(s)\Big|_{s=(\Lambda_R^j)^2} = \frac{(F_n^j)^2 (\Lambda_R^j)^2}{a_j} \simeq N_c C_0 (\Lambda_{PT}^j)^2. \tag{12}$$

A more precise quantitative estimation of $(\Lambda_R^j)^2$ for a fixed ϵ is difficult as the additive Breit-Wigner description of individual resonances is not reliable when there is substantial overlap. Nevertheless a semi-quantitative estimate can be done: let us determine N^j by demanding that oscillations due to resonances relative to the background be $\sim \epsilon$. The value of the minimum between two adjacent resonances (in the Breit-Wigner approximation) is reached for $s \simeq \frac{1}{2}\Big((m_{n-1}^j)^2 + (m_n^j)^2\Big)$, and for $n \simeq N^j$

$$\pi\rho_n^j(s)\Big|_{n=N^j} \simeq \frac{(F^j)^2 (N^j)^2 \frac{B^j}{N_c}}{\frac{1}{4} + \frac{(N^j B^j)^2}{N_c^2}}. \tag{13}$$

Comparing this with (11)), we get $N^j \simeq N_c / 2\sqrt{\epsilon} B_j$.

The Regge model also implies asymptotically equal decay constants; that is $F_n^j \sim F^j$, $n \gg 1$. From the previous arguments $(F^j)^2 \simeq N_c C_0 a_j$. Assuming that $(\Lambda^V)^2 - (\Lambda^A)^2$ is at most of $\mathcal{O}(1)$ in the large N_c expansion, the Weinberg sum rules lead immediately to the conclusion that $F^V \simeq F^A$ because otherwise in the first Weinberg sum rule (8)

$$\sum_{n=0}^{N^V} (F_n^V)^2 - \sum_{n=0}^{N^A} (F_n^A)^2 \sim N_c \Big((F^V)^2 - (F^A)^2\Big) \sim N_c^2$$

whereas it should be of $\mathcal{O}(N_c)$.

Next let us analyze the linear Regge asymptotics for radial excitations. Consider meson states lying on the trajectories asymptotically, $(m_n^j)^2 \simeq (m_0^j)^2 + a^j n$, $n \gg 1$. Then, if $F^{V,A} \simeq F$, from the second Weinberg sum rule (9) we get $a^V \simeq a^A \equiv a$, *i.e.* the slope of trajectories is universal. Indeed, let us write $(\Lambda^j)^2 \simeq N^j a^j + (\Lambda_0^j)^2$.

Then up to terms subleading in N_c

$$\Big[(F^V)^2\Big(\frac{1}{2}N^V(N^V+1)a^V + N^V(m_0^V)^2\Big) \\ -(F^A)^2\Big(\frac{1}{2}N^A(N^A+1)a^A + N^A(m_0^A)^2\Big)\Big] \\ \simeq \frac{1}{2}\Big[(N^V)^2(F)^2(a^V-a^A)\Big] \tag{14}$$

vs.

$$\frac{1}{2}N_cC_0\left[\Big(N^Va^V+(\Lambda_0^V)^2\Big)^2-\Big(N^Aa^A+(\Lambda_0^A)^2\Big)^2\right] \\ \simeq \frac{1}{2}N_cC_0(N^V)^2\Big((a^V)^2-(a^A)^2\Big),$$

which match each other iff $a^V=a^A$ for $(F^j)^2 \simeq N_cC_0a_j$.

Finally, from (14) and from the relation $F^2 = N_cC_0a$ it follows that

$$(m_0^V)^2-(m_0^A)^2 \simeq (\Lambda_0^V)^2-(\Lambda_0^A)^2 \tag{15}$$

at leading order. Indeed, the second sum rule is saturated by $N^VF^2\Big((m_0^V)^2-(m_0^A)^2\Big) \simeq N^VN_cC_0a\Big((\Lambda_0^V)^2-(\Lambda_0^A)^2\Big)$ whereas other terms are finite. Thus a possible finite shift between mass spectra of mesons with different parities in the large N_c approximation has to be accompanied with the same shift in cutoffs for resonance regions, even though as we have argued we expect this difference to be subleading in N_c. However a deviation from the universality $a^V=a^A$ may also give a comparable term.

We stress once more that the results are not based on the numbers of resonances N^V, N^A or cutoffs Λ_R^V,Λ_R^A being equal. We do expect however that their difference is subleading in N_c for the reasons given above.

5. To conclude: For large N_c the region of transition from the resonance dominated region to the perturbatively dominated one is shrinking. The cutoffs that separate these two regions then: (a) may not coincide in opposite parity channels exactly; (b) their values squared grow with N_c and their difference is subdominant in the large N_c expansion. In this case, for linear Regge trajectories: (1) the ratio of asymptotic widths to masses of of resonances is the same in opposite-parity channels; (2) the asymptotics of decay coupling constants as well as the Regge slopes coincide for opposite parity channels; (3) the cutoffs in opposite parity channels don't coincide, but the Regge slopes are universal, and the Regge trajectory intercepts differ in a gap which is fully determined by the difference in the above cutoffs. Similar arguments can be applied to other pairs of channels with the equal quantum numbers but parity.

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